Efficient Sidelobe ASK Based Dual-Function Radar-Communications

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ABSTRACT

Recently, dual-function radar-communications (DFRC) has been proposed as means to mitigate the spectrum congestion problem. Existing amplitude-shift keying (ASK) methods for information embedding do not take full advantage of the highest permissable sidelobe level. In this paper, a new ASK-based signaling strategy for enhancing the signal-to-noise ratio (SNR) at the communication receiver is proposed. The proposed method employs one reference waveform and simultaneously transmits a number of orthogonal waveforms equals to the number of 1's in the binary sequence being embedded. 3 dB SNR gain is achieved using the proposed method as compared to existing sidelobe ASK methods. The effectiveness of the proposed information embedding strategy is verified using simulations examples.

1. INTRODUCTION

In recent years, the problem of radio frequency spectrum congestion has been the focus of intensive research [1]–[8]. Specifically, the coexistence of radar and communications has attracted a lot of interest [9]–[24]. The joint operation of radar and communications using dual-functionality platforms has been recently proposed as an effective tool to ease competition over bandwidth. Several signaling strategies for information embedding into the radar emission have been reported in the literature including the waveform diversity based scheme [11]–[14], the sidelobe amplitude-modulation based scheme [16], the amplitude-shift keying (ASK) based scheme [18], [19], and the phase-shift keying (PSK) based scheme [21], [23], [24]. Some of these schemes require the realization of multiple orthogonal waveforms. The problem of waveform design with good auto- and cross-correlation properties has been investigated thoroughly in multiple-input multiple-output radar [25]–[32], cognitive radar [33], and spectral coexistence [34]–[37].

In this paper, we consider the problem of sidelobe control for dual-function radar-communication (DFRC) systems. We develop a new method that enables information embedding into the radar-emission via modulating the sidelobe of the instantaneous beampattern. The radar operation usually dictates that the leakage of the transmit power within the sidelobe region be minimized. Therefore, the number of distinct sidelobe levels that can be used to reliably deliver information is small. The state of the art methods for DFRC employ sidelobe ASK and multiple orthogonal waveforms to embed one bit of information per waveform. However, dividing the transmitted power evenly among the waveforms being used yields low signal-to-noise ratio (SNR) at the communication receiver. We propose a new signaling strategy for enhancing the SNR leading to an improvement in detection performance at the communication receiver. The proposed method employs one reference waveform and simultaneously transmits a number of orthogonal waveforms equals to the number of 1's in the binary sequence being embedded. The proposed method enables achieving 3 dB SNR gain as compared to existing sidelobe ASK methods. In addition, the employed waveform diversity can be used to enhance the radar performance. The effectiveness of the proposed information embedding strategy is verified using simulations examples.

2. SIGNAL MODEL

Consider a DFRC system equipped with joint transmit platform, a radar receive array, and a single-element communication receiver. The joint transmit platform consists of M transmit arranged in an arbitrary linear shape. The purpose of the dual-function joint transmit array is to embed information toward the direction of the communication receiver as a secondary function to the primary radar function of the DFRC system. The joint transmit array is used to focus the transmit power within the main radar beam while permitting variation in the sidelobe level (SLL) towards the communication direction. The $M \times 1$ vector of the baseband signals at the input of the joint transmit platform is given by

$$\mathbf{s}(t) = \sum_{k=1}^{K} \mathbf{w}_k^* \psi_k(t), \tag{1}$$

where t denotes the fast-time index, $\psi_k(t),\ k = 1, \ldots, K$ are K orthogonal waveforms, $\mathbf{w}_k,\ k = 1, \ldots, K$ are the $M \times 1$ transmit beamforming weight vectors, and $(\cdot)^*$ denotes the complex conjugate. The waveforms $\psi_k(t),\ k = 1, \ldots, K$ are assumed to satisfy the orthogonality condition at zero time-delay, that is $\int_{\mathrm{T_p}} \psi_k(t)\psi_{k'}^*(t) = 0,\ k \neq k'$, where $\mathrm{T_p}$ is the pulse width. At the communication receiver, the orthogonal waveforms are used for matched-filtering enabling the extraction of the received signals components associated with each transmitted waveform. The transmit beamforming weight vectors $\mathbf{w}_k,\ k = 1, \ldots, K$, are appropriately designed to focus the transmit power within the main beam of the radar. In addition, the transmit beamforming weight vectors should achieve certain pre-determined SLLs towards the communication direction. The latter feature enables the embedding of binary information into the radar emission as will be explained in subsequent sections in the paper.

Consider L far-field targets that are located in a certain range-bin. The radar receive array is assumed to comprise N colocated receive elements arranged in an arbitrary shape. The $N \times 1$ vector of baseband signals received by the radar during a certain radar pulse is expressed as

$$\mathbf{x}(t) = \sum_{m=1}^{L} \beta_m \Big(\mathbf{a}^T(\theta_m)\mathbf{s}(t)\Big)\mathbf{b}(\theta_m) + \check{\mathbf{x}}(t) + \mathbf{z}(t), \tag{2}$$

where β_m is the reflection coefficient of the mth target which obeys the Swerling II target model, i.e., the reflectivity remains constant during the entire radar pulse but changes from pulse to pulse, $\mathbf{a}(\theta_m)$ and $\mathbf{b}(\theta_m)$ are the $M \times 1$ and the $N \times 1$ steering vectors in direction θ_m of the transmit and receive arrays, respectively, $\check{\mathbf{x}}(t)$ is the $N \times 1$ vector comprises the signals that impinge on the receive array from the sidelobe region, $\mathbf{z}(t)$ is the $N \times 1$ vector of additive white Gaussian noise with zero mean and covariance $\sigma_z^2\mathbf{I}_N$, $(\cdot)^T$ denotes matrix transpose, and $\mathbf{I}_N$ is the $N \times N$ identity matrix. It is worth noting that the processing of the radar received data can be performed directly on the $N \times 1$ data vector $\mathbf{x}(t)$, i.e., without making use of the waveform diversity. However, it is possible for the radar to employ the waveform diversity by requiring the transmit waveforms to be orthogonal at all time-delays and Doppler-shifts within the range and velocity specifications of the radar. In practice, it is difficult to synthesize perfectly orthogonal waveforms with overlapped spectral contents. Alternatively, waveforms with low cross-correlations can be used in practice. The design of waveforms with low cross-correlations properties has been extensively investigated in the literature (see [35]–[37], and references therein).

Consider a single-element communication receiver located at an arbitrary direction θ_c within the sidelobe region. The orthogonal waveform dictionary used at the transmitter is assumed to be known to the communication receiver. The baseband signal at the output of the communication receiver is given as

$$\begin{aligned} c(t) &= \alpha_{\mathrm{ch}}\Big(\mathbf{a}^T(\theta_c)\mathbf{s}(t)\Big) + n_c(t) \\ &= \alpha_{\mathrm{ch}}\sum_{k=1}^{K}\mathbf{w}_k^H\mathbf{a}(\theta_c)\psi_k(t), \end{aligned} \tag{3}$$

where α_{ch} is the channel coefficient which summarizes the propagation environment between the transmit array and the communication receiver and $n_c(t)$ is the additive white Gaussian noise with zero mean and variance σ_c^2.

3. MULTI-WAVEFORM ASK-BASED INFORMATION EMBEDDING

In this section, we first briefly review existing ASK-based information embedding scheme reported in [19] and discuss its power efficiency in the sidelobe region. Then, we present our proposed efficient ASK-based scheme and highlight its advantages.

3.1 Existing ASK-based Scheme

The multi-waveform ASK method of [19] employs multiple waveforms and two transmit beamforming weight vectors denoted as $\mathbf{u}_{\mathrm{H}}$ and $\mathbf{u}_{\mathrm{L}}$. The corresponding signaling strategy is illustrated in Fig. 1. Let Δ_{H} and Δ_{L} be the SLLs associated with $\mathbf{u}_{\mathrm{H}}$ and $\mathbf{u}_{\mathrm{L}}$ in the spatial direction of the communication receiver, respectively. We assume that the condition $\Delta_{\mathrm{H}} > \Delta_{\mathrm{L}}$ is satisfied while designing the vectors $\mathbf{u}_{\mathrm{H}}$ and $\mathbf{u}_{\mathrm{L}}$.

In order to embed K bits per radar pulse, K waveforms are transmitted simultaneously where the total transmit power P is divided equally among the K waveforms. Every transmitted waveform is used to deliver one information bit to the communication receiver. During each radar pulse, the waveform $\psi_n(t),\ n = 1, \ldots, K$, is radiated either via $\mathbf{u}_{\mathrm{H}}$ for $b_k = 0$ or $\mathbf{u}_{\mathrm{L}}$ when $b_k = 1$. Therefore, the transmit signals (1) can be rewritten as

$$\mathbf{s}(t) = \sqrt{\frac{P}{K}}\sum_{k=1}^{K}\Big(b_k\mathbf{u}_{\mathrm{L}}^* + (1-b_k)\mathbf{u}_{\mathrm{H}}^*\Big)\psi_k(t). \tag{4}$$

The communication receiver signal model (3) is rewritten as

$$\begin{aligned} y_c(t) &= \sqrt{\frac{P}{K}}\alpha_{\mathrm{ch}}\sum_{k=1}^{K}\Big(b_k\mathbf{u}_{\mathrm{L}}^H\mathbf{a}(\theta_c) + \big(1-b_k\big)\mathbf{u}_{\mathrm{H}}^H\mathbf{a}(\theta_c)\Big)\psi_k(t) + \mathbf{n}_c(t) \\ &= \sqrt{\frac{P}{K}}\alpha_{\mathrm{ch}}\sum_{k=1}^{K}\Big(b_k\Delta_{\mathrm{L}} + (1-b_k)\Delta_{\mathrm{H}}\Big)\psi_k(t) + n_c(t). \end{aligned} \tag{5}$$

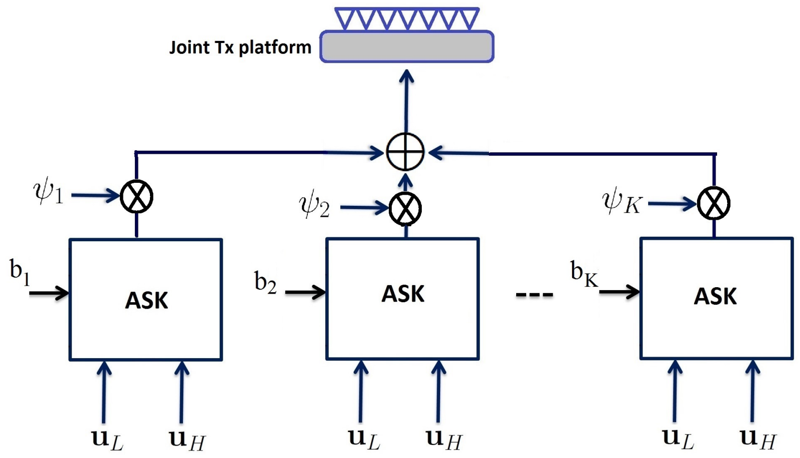

Figure 1. ASK-based signaling for information embedding; method of [19].

The embedded binary bits can be extracted from the received signal component associated with each transmitted orthogonal waveform using matched filtering. The kth matched filter output y_k, $k = 1, \dots, K$, can be expressed as

$$y_k = \begin{cases} \sqrt{\frac{P}{K}}\alpha_{\text{ch}}\Delta_{\text{H}} + n_k, & b_k = 0, \\ \\ \sqrt{\frac{P}{K}}\alpha_{\text{ch}}\Delta_{\text{L}} + n_k, & b_k = 1, \end{cases} \tag{6}$$

where n_k is the additive noise term which has the same statistics as that of $n_c(t)$.

Measuring the signal strength at the output of each matched filter, the transmitted bits can be extracted by performing the following ratio test

$$\hat{b}_k = \begin{cases} 0, & \text{if } |y_k| \geq T_0, \\ \\ 1, & \text{if } |y_k| < T_0, \end{cases} \tag{7}$$

where T_0 is a threshold. Note that the embedding and detection of each bit are performed independently from other bits due to the orthogonality of the transmitted waveforms.

3.2 Proposed ASK-based Scheme

Assuming that the probability of transmitting $b_k = 0$ equals the probability of transmitting $b_k = 1$, the ASK signaling scheme presented in Section on average embeds half of the binary data via high SLL Δ_H while the other half is embedded via low SLL Δ_L. Embedding information via Δ_L means that the secondary communication function does not take full advantage of the highest SLL permissable by the primary radar function of the joint system. To enhance the power efficiency towards the communication direction, we propose to transmit the waveforms associated with binary data 1's while withholding the waveforms associated with 0's. During each radar pulse, the transmitted waveforms are radiated via Δ_H. This results in enhancing the power efficiency towards the communication direction by using the highest SLL allowed by the radar. This signaling scheme is illustrated in Fig. 2

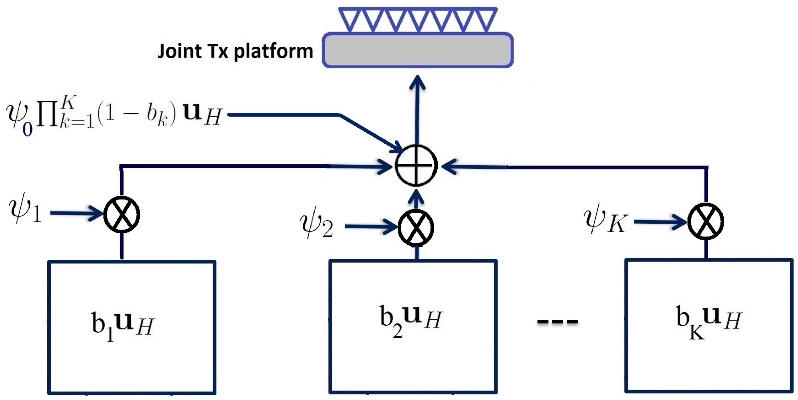

Figure 2. ASK-based signaling for information embedding.

Assume that during a certain radar pulse the number of 1's in the binary message is $\tilde{K}$ while the number of 0's is $K-\tilde{K}$. In this case, the baseband representation of the transmitted signals is given by

$$\mathbf{s}(t)=\sqrt{P}\Big(\prod_{k=1}^{K}(1-b_k)\Big)\mathbf{u}_H^{*}\psi_0(t)+\sqrt{\frac{P}{\tilde{K}}}\sum_{k=1}^{K}b_k\mathbf{u}_H^{*}\psi_k(t). \tag{8}$$

The first term in the right hand side of (8) is radiated in the special case when the entire message consists of all 0's.

The communication receiver signal is given by

$$\begin{aligned} y_c(t)&=\sqrt{P}\alpha_{\mathrm{ch}}\Big(\prod_{k=1}^{K}(1-b_k)\Big)\mathbf{u}_H^{H}\mathbf{a}(\theta_c)\psi_0(t)+\sqrt{\frac{P}{\tilde{K}}}\alpha_{\mathrm{ch}}\sum_{k=1}^{K}b_k\mathbf{u}_{\mathrm{H}}^{H}\mathbf{a}(\theta_c)\psi_k(t)\\ &=\sqrt{P}\alpha_{\mathrm{ch}}\Big(\prod_{k=1}^{K}(1-b_k)\Big)\Delta_H+\sqrt{\frac{P}{\tilde{K}}}\alpha_{\mathrm{ch}}\sum_{k=1}^{K}b_k\Delta_{\mathrm{H}}\psi_k(t)+n_c(t). \end{aligned} \tag{9}$$

The embedded binary bits can be extracted from the received signal component associated with each transmitted orthogonal waveform using matched filtering. The kth matched filter output $y_k,\ k=1,\ldots,K$, can be expressed as

$$y_k=\begin{cases}\sqrt{\frac{P}{\tilde{K}}}\alpha_{\mathrm{ch}}\Delta_{\mathrm{H}}+n_k, & b_k=1,\\ n_k, & b_k=0.\end{cases} \tag{10}$$

Measuring the signal strength at the output of each matched filter, the transmitted bits can be extracted by performing the following ratio test

$$\hat{b}_k=\begin{cases}1, & \text{if } |y_k|\geq T_0,\\ 0, & \text{if } |y_k|<T_0.\end{cases} \tag{11}$$

It is worth noting that, on average, $\tilde{K}=K/2$. Therefore, the power assigned to each waveforms in (8) is twice that of the corresponding waveform in the signal model (4). This results in 3 dB SNR gain resulting in improvement in communication detection performance.

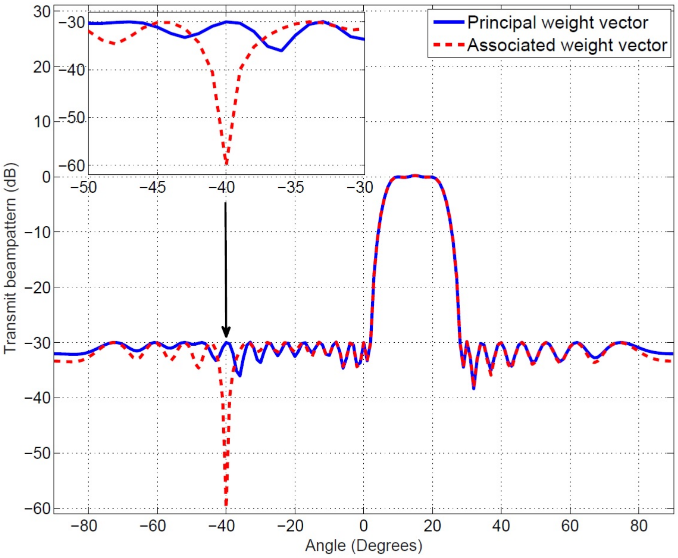

Figure 3. Transmit beampattern versus angle.

4. SIMULATION RESULTS

In the simulation, we consider a uniform linear transmit array consisting of $M = 25$ antennas spaced one-half wavelength apart. The main radar operation takes place within the sector $\mathbf{\Theta} = [10° \; 20°]$. Three examples are considered. A single communication direction towards the spatial direction $\theta_c = -40°$ is assumed. We use the method in [19] to design the principal radar transmit weight vector $\mathbf{w}_0$ and the associated weight vector $\mathbf{w}_1$ required to achieve bi-level sidelobes towards the communication direction. Sophisticated methods for designing transmit beamforming weight vectors which desirable mainlobe and sidelobe specifications can be borrowed from the literature (see [38] and [39]; and references therein). Fig. 3 shows the corresponding transmit beampatterns It is clear from the figure that the sidelobe attenuation with respect to the main beam is larger than 30 dB for all directions within the out-of-sector region. The figure also shows that the SLL towards the communication direction attains the maximum value allowable to ensure that the communication receiver receives the highest possible power within the sidelobe of the radar.

In the second example, we test the BER performance versus SNR. To compute the BER, 10^6 pulses are considered, i.e., the process of information embedding is repeated independently for 10^6 times. During each radar pulse, 1 bit, 4 bits, 16 bits, and 64 bits are embedded. The propagation coefficient α_c is modeled as a random variable with unit magnitude and uniformly distributed random phase. Fig. 4 shows the BERs for the exiting and the proposed ASK-based methods tested versus SNR. The figure shows that the BER curves for the proposed method are superior to the BER curves for the existing sidelobe ASK method.

In the third example, we test the BER performance versus angle while fixing the SNR. We repeat the second example by fixing the SNR to 25 dB while varying the communication receiver direction. Figure 5 shows the BER curves versus the angle of the communication direction for the proposed and the existing ASK methods.

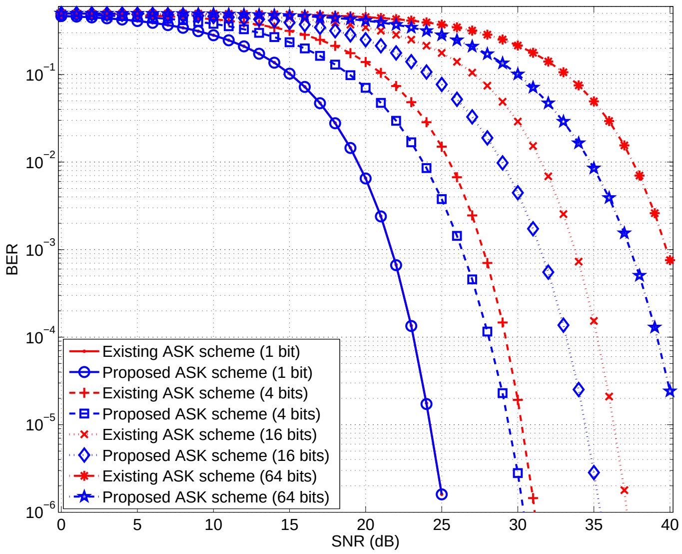

Figure 4. BER versus SNR.

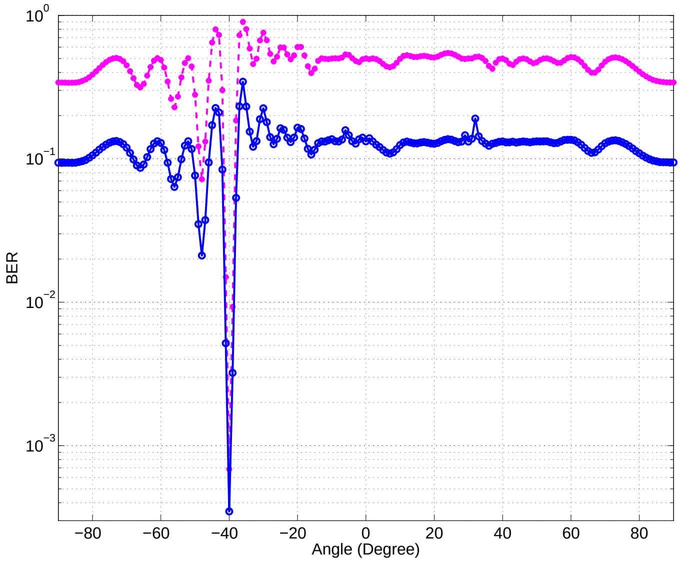

Figure 5. BER versus angle.

The figure that that both methods enable communication delivery towards the intended communication direction only. It is clear from the figure that eavesdroppers located at directions other than the intended communication direction cannot detect the embedded data reliably. The figure also shows that the BER performance for the proposed scheme is superior to the performance of the existing ASK scheme.

In the last example, we test the radar receiver performance for DFRC. We use the same setup as in the previous examples but we assume two interference targets located in the sidelobe region at directions -40° and -39°, respectively. The directions of the interfering targets are estimated using data collected from 100

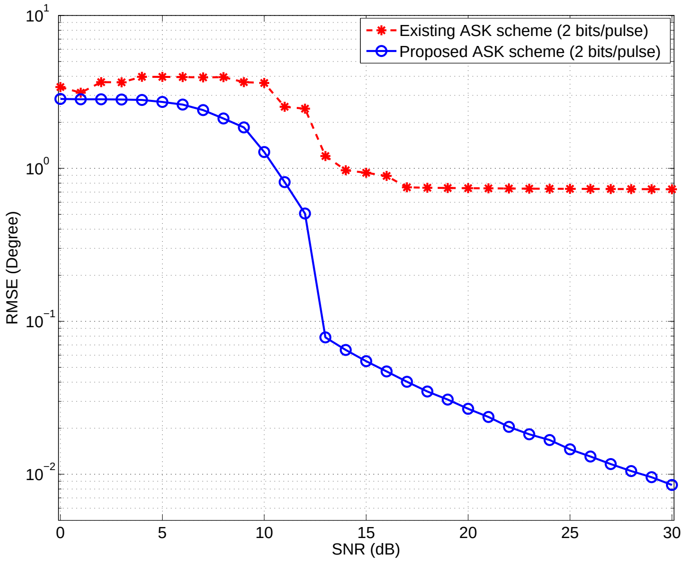

Figure 6. RMSE versus SNR.

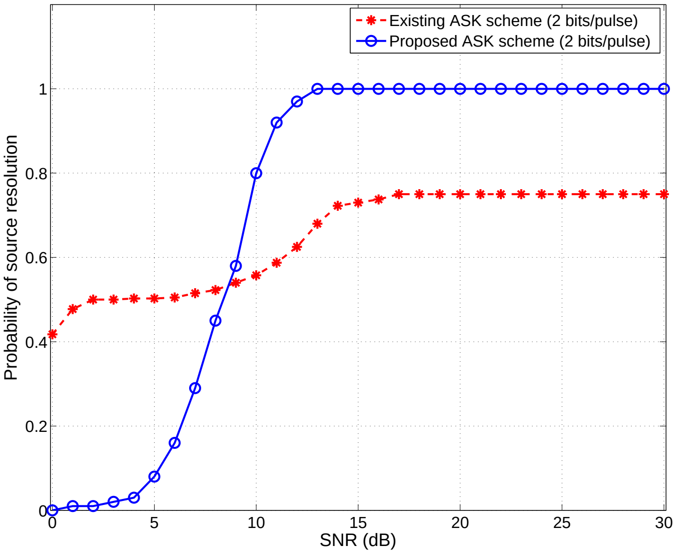

Figure 7. Probability of source resolution versus SNR.

pulses (i.e., 100 data snapshots). The spectral MUSIC algorithm is used to estimate the targets directions. Figures 6 and 7 show the root mean-square error (RMSE) and the probability of target resolution versus SNR, respectively. The two figures show that the radar receiver performance is superior when the proposed signaling scheme is used.

5. CONCLUSIONS

The problem of dual-function radar-communication (DFRC) system design was considered. A new signaling strategy for enhancing the signal-to-noise ratio (SNR) at the communication receiver was proposed. The proposed method employs one reference waveform and simultaneously transmits a number of orthogonal waveforms equals to the number of 1's in the binary sequence being embedded. It was shown that 3 dB SNR gain can be achieved using the proposed method as compared to existing sidelobe ASK methods. The effectiveness of the proposed information embedding strategy was verified using simulations examples.

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