Computationally Efficient Beampattern Synthesis for Dual-Function Radar-Communications

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ABSTRACT

The essence of amplitude-modulation based dual-function radar-communications is to modulate the sidelobe of the transmit beampattern while keeping the main beam, where the radar function takes place, unchanged during the entire processing interval. The number of distinct sidelobe levels (SLL) required for information embedding grows exponentially with the number of bits being embedded. We propose a simple and computationally cheap method for transmit beampattern synthesis which requires designing and storing only two beamforming weight vectors. The proposed method first designs a principal transmit beamforming weight vector based on the requirements dictated by the radar function of the DFRC system. Then, a second weight vectors is obtained by enforcing a deep null towards the intended communication directions. Additional SLLs can be realized by simply taking weighted linear combinations of the two available weight vectors. The effectiveness of the proposed method for beampattern synthesis is verified using simulations examples.

1. INTRODUCTION

The problem of beampattern synthesis has been thoroughly investigated for several decades in many diverse fields such as radar, sonar, wireless communications, radio astronomy, seismology, and other other applications [1]. Adaptive and robust beamforming in passive receive arrays have received a lot of interest over the years [1]–[3]. The last decade witnessed the development of multiple-input-multiple-output (MIMO) radar which offers better resolution and higher degrees of freedom as compared to conventional phased-array radar [4]–[10]. Several transmit beamforming techniques have been developed to achieve transmit coherent processing gain in MIMO radar [11]–[14]. These transmit beamforming techniques design the transmit beampattern such that the radar specifications are satisfied.

Recently, the problem of radio frequency spectrum congestion has attracted a lot of interest [15]–[19]. In particular, the coexistence of radar and communications has been the focus of intensive research [20]–[28]. The emerging concept of dual-function radar-communications (DFRC) has been recently proposed as an effective mean to ease competition over bandwidth. Signaling strategies for DFRC include the sidelobe amplitude-modulation technique [29], the sidelobe amplitude-shift keying (ASK) based technique [30], and the phase-shift keying (PSK) based technique [31]. These techniques enforce additional constraints to the beampattern synthesis problem to enable the embedding of communication symbols into the radar emission. Therefore, innovative and efficient beampattern synthesis techniques for DFRC are needed.

In this paper, we propose a simple and computationally cheap method for transmit beampattern synthesis which requires designing and storing only two beamforming weight vectors. The proposed method first designs a principal transmit beamforming weight vector based on the requirements dictated by the radar function of the DFRC system. Then, a second weight vectors is obtained via minimizing the difference between the second weight vector and the principal one while enforcing a deep null towards the intended communication direction. Convex optimization based formulations are used to obtain the two weight vectors. An arbitrary number of beampattern with district SLLs is realized by simply taking weighted linear combinations of the two available weight vectors. The effectiveness of the proposed method for beampattern synthesis is verified using simulations examples.

The paper is organized as follows. Section 2 gives the signal model of the dual-functionality system. Section 3 provides a concise review of estimating beampattern synthesize methods for DFRC and presents our proposed synthesize method. Simulation results are given in Section 4. Finally, conclusions are drawn in Section 5.

2. SIGNAL MODEL

Consider a DFRC system equipped with dual-functionality transmit platform, a radar receive array, and a single-element communication receiver. The transmit platform consists of M transmit antennas arranged in an arbitrary linear shape. We assume that transmit platform and the radar receive array are closely spaced to each other such that a target located in the far-field would be at the same spatial angle with respect to both arrays. The dual-functionality transmit platform is used to focus the transmit power within the main radar beam while permitting variation in the sidelobe level (SLL) towards the communication direction. The $M \times 1$ vector of the baseband signals at the input of the joint transmit platform is given by

$$\mathbf{s}(t) = \mathbf{w}^* \psi(t), \tag{1}$$

where t denotes the fast-time index, $\psi(t)$ is the radar waveform, $\mathbf{w}$ is the $M \times 1$ transmit beamforming weight vector, and $(\cdot)^*$ denotes the complex conjugate. The transmit beamforming weight vector $\mathbf{w}$, is appropriately designed to focus the transmit power within the main beam of the radar. In addition, the transmit beamforming weight vector should enable modulating the sidelobe levels (SLLs) towards the communication direction. The latter feature enables the embedding of communication symbols into the radar emission.

Consider a single-element communication receiver located at an arbitrary direction θ_c within the sidelobe region. The waveform used at the dual-functionality transmit platform is assumed to be known to the communication receiver. The baseband signal at the output of the communication receiver is given as

$$\begin{aligned} c(t) &= \alpha_{\text{ch}} \Big(\mathbf{a}^T(\theta_c) \mathbf{s}(t) \Big) + n_c(t) \\ &= \alpha_{\text{ch}} \mathbf{w}^H \mathbf{a}(\theta_c) \psi(t), \end{aligned} \tag{2}$$

where α_{ch} is the channel coefficient which summarizes the propagation environment between the transmit array and the communication receiver, $n_c(t)$ is the additive white Gaussian noise with zero mean and variance σ_c^2, and $(\cdot)^H$ denotes the Hermitian operation.

3. SIDELOBE AM BASED INFORMATION EMBEDDING

The essence of the recent sidelobe AM based technique is to embed communication symbols into the radar emission via modulating the SLL towards the intended communication direction [29]. The primary radar operation requirements are satisfied by keeping the radar mainlobe unchanged during the entire coherent processing interval (CPI). We assume that Q bits of binary information need to be embedded during each radar pulse (or a time interval of pre-specified duration). The Q bits are first mapped into a dictionary of $K = 2^Q$ amplitude symbols denoted as $\mathbb{D}_{\mathrm{AM}} = \{\Delta_1, \ldots, \Delta_K\}$. Each symbol Δ_k can be represented by a certain pre-specified SLL. Therefore, in order to implement this information embedding technique, a single radar waveform and K distinct SLLs are required.

It is worth noting that the data rate can be increased by employing multiple orthogonal waveforms where each waveforms can be used to embed Q bits during each time interval. However, in practice, it could be difficult to realize perfectly orthogonal waveforms. In this case, waveforms with low cross-correlations can be realized. Waveforms with low cross-correlations properties cab ne designed using several recently developed methods see [32]–[37]. The number of bits that can be simultaneously embedded during each pulse becomes Q times the number of orthogonal waveforms used. In this paper, we consider the single waveforms case only.

In the following, we summarize two approaches to achieve the required K number of distinct SLLs. We also introduce an efficient method for synthesizing the required K beampatterns.

3.1 Sidelobe Modulation Using Time-Modulated

Time-modulated array can be used to optimize the average array factor within the entire CPI, i.e., within the radar integration time as suggested in [29]. This method divides the radar integration time into P time intervals. The average AF for a ULA is expressed as [29]

$$\mathrm{AF} = \frac{1}{P} \sum_{p=1}^{P} \sum_{m=0}^{M-1} w_{m,p} \exp\left\{ -j \frac{2\pi}{\lambda} d \sin\theta_t \right\}, \tag{3}$$

where $w_{m,p}$ denotes the complex weight associated with the mth transmit antenna during the pth time interval, λ denotes the propagation wavelength, and θ_t is the spatial direction of the main radar beam. Two methods are reported in [29] for solving the problem of designing the weights. The first method uses a sparse time modulated array. This has the advantage of simple implementation via switching the antennas on and off. However, the sparsity of the array results in reducing degrees of freedom and, therefore, sparse time modulated array is capable of achieving few distinct SLLs which limits the number of communication symbols that can be embedded. The second method uses phase-only weights, i.e., it fixes the amplitude of the complex weights and optimizes the AF over the phase parameters. In both of the above methods, the AF optimization involves a non-convex highly non-linear optimization function which is difficult to solve in a computationally efficient manner.

3.2 Beampattern Synthesis Using Convex Optimization

Convex optimization based beampattern synthesis can be used to achieve K distinct SLLs via designing K transmit beamforming weight vectors [26], [30]. The SLL associated with each transmit weight vector is then used to represent one communication symbol. The design of the K weight vectors should take into account the primary radar operation requirements such as keeping the transmit beampattern within the main beam of the radar unchanged during the entire CPI. Assuming that the radar operation takes place in a narrow mainbeam, i.e., the radar needs to focus the transmit power towards the spatial angle θ_t. One way to design each of the required K transmit beamforming weight vectors, denoted as $\mathbf{u}_k,\ k = 1, \ldots, K$, is to minimize the power radiation level in the sidelobe region $\bar{\boldsymbol{\Theta}}$, while maintaining a distortionless response towards the desired direction θ_t. In addition, constraints on the SLLs towards the intended communication directions can be enforced to achieve sidelobe modulation. Without loss of generality, we assume a single communications receiver located at direction θ_c. Then, the transmit beamforming design problem can be formulated as the following optimization problem [26]

$$\begin{aligned} \min_{\mathbf{u}_k} \quad \max_{\theta} \quad & \left|\mathbf{u}_k^H \mathbf{a}(\theta)\right|, \quad \theta \in \bar{\boldsymbol{\Theta}}, && (4)\\ \text{subject to} \quad & \mathbf{u}_k^H \mathbf{a}(\theta_t) = 1, && (5)\\ & \mathbf{u}_k^H \mathbf{a}(\theta_c) = \Delta_k, && (6) \end{aligned}$$

where Δ_k is a pre-determined positive number of user choice which can be used to determine the amount of transmit power radiated towards the communications directions over the kth transmit beam and $|\cdot|$ denotes the absolute value of a complex number. It is worth noting that the optimization problem (4)–(6) is convex. therefore, it can be efficiently solved using the interior point methods [38]. However, the aforementioned optimization problem needs to be solved K times, i.e., it should be solved separately for each transmit beamforming weight vector $\mathbf{u}_k,\ k = 1, \ldots, K$.

3.3 Proposed Efficient Beampattern Synthesis

It is worth noting that the convex optimization based beampattern synthesis described in Sec. 3.2 becomes computationally prohibitive for the case when the number of bits Q that need to be embedded is large. In this case, the number of beampatterns K that need to be synthesized becomes very large and, therefore, it becomes computationally demanding to solve the optimization problem (4)–(6) K times. Moreover, in situations where the either the joint transmit platform or the communication receiver is not stationary, the entire set K beampatters have to be redesigned for whenever the relative angle between the joint transmit platform and the communication receiver changes. This makes it computationally very demanding and limits its practical application. Here, we propose a simple and efficient way to design the K number of transmit beampatters. The proposed method requires solving tow optimization problems to obtain two transmit beamforming weight vectors. Then, it uses a linear combination of the two available weight vectors to obtain an arbitrary number of weight vectors with distinct SLLs.

Assume that the principal transmit beamforming weight vector $\mathbf{w}$ which satisfy all requirements mandated by the primary radar operation is given. For example, it can be obtained as the solution to the following convex optimization problem

$$\min_{\mathbf{w}} \quad \max_{\theta} \quad \left|\mathbf{w}^H \mathbf{a}(\theta)\right|, \quad \theta \in \bar{\boldsymbol{\Theta}}, \tag{7}$$

$$\text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_t) = 1. \tag{8}$$

It is worth noting that if the radar needs to operate in a phased-array mode, then the principal weight vector should be taken as $\mathbf{w} = \mathbf{a}^*(\theta_t)$.

The principal weight vector $\mathbf{w}$ can be used to design an associated weight vector $\tilde{\mathbf{w}}$ which has almost the same transmit radiation beampattern as that of $\mathbf{w}$ except that it enforces deep nulls towards the intended communication direction. One way to design the associated weight vector $\tilde{\mathbf{w}}$ is to minimize to difference between $\tilde{\mathbf{w}}$ and $\mathbf{w}$ while enforcing the null constraint towards the intended communication direction, that is,

$$\min_{\tilde{\mathbf{w}}} \quad \left\|\mathbf{w} - \tilde{\mathbf{w}}\right\| \tag{9}$$

$$\text{subject to} \quad \tilde{\mathbf{w}}^H \mathbf{a}(\theta_c) = 0, \tag{10}$$

$$\tilde{\mathbf{w}}^H \mathbf{a}(\theta_t) = M, \tag{11}$$

where $\|\cdot\|$ denotes the Euclidean norm. Note that the constraint (10) enforces a null towards the intended communication direction while the constraint (11) ensures that both $\tilde{\mathbf{w}}$ and $\mathbf{w}$ have the same transmit gain towards the direction θ_t.

Once the principal and the associated weight vectors $\tilde{\mathbf{w}}$ and $\mathbf{w}$ are obtained, the K required beampatterns can be obtained by taking the corresponding weight vectors $\mathbf{u}_k, \; k = 1, \ldots, K$ as

$$\mathbf{u}_k = \eta_1 \mathbf{w} + \eta_2 \tilde{\mathbf{w}}, \tag{12}$$

where $\eta_1 = \frac{K-k+1}{K}$ and $\eta_2 = \frac{k-1}{K}$. it is worth noting that for $k = 1$, (12) simplifies to $\mathbf{u}_1 = \mathbf{w}$.

4. SIMULATION RESULTS

In the simulation, we consider a uniform linear transmit array consisting of $M = 20$ antennas spaced one-half wavelength apart. The main radar operation takes place within a narrow beam focused at $\theta_t = 0°$. We assume a single communication direction towards the spatial direction $\theta_c = -40°$. We solve the optimization problem (10)–(11) to design the principal radar transmit weight vector $\mathbf{w}$ and the associated weight vector $\tilde{\mathbf{w}}$. Fig. 1 shows the corresponding transmit beampatterns. It is clear from the figure that the sidelobe attenuation with respect to the main beam is in the range of 30 dB for all directions within the out-of-sector region. The figure also shows that the SLL towards the communication direction attains the maximum value allowable for the principal weight vector and has a deep null for the associated weight vector. The two weight vectors are used to synthesize five beampatterns using the simple formula (12). The corresponding beampatterns are plotted in Fig. 2. It is clear from the figure that all five beampatterns are almost the same except at the communication direction where each beampattern has a distinct SLL.

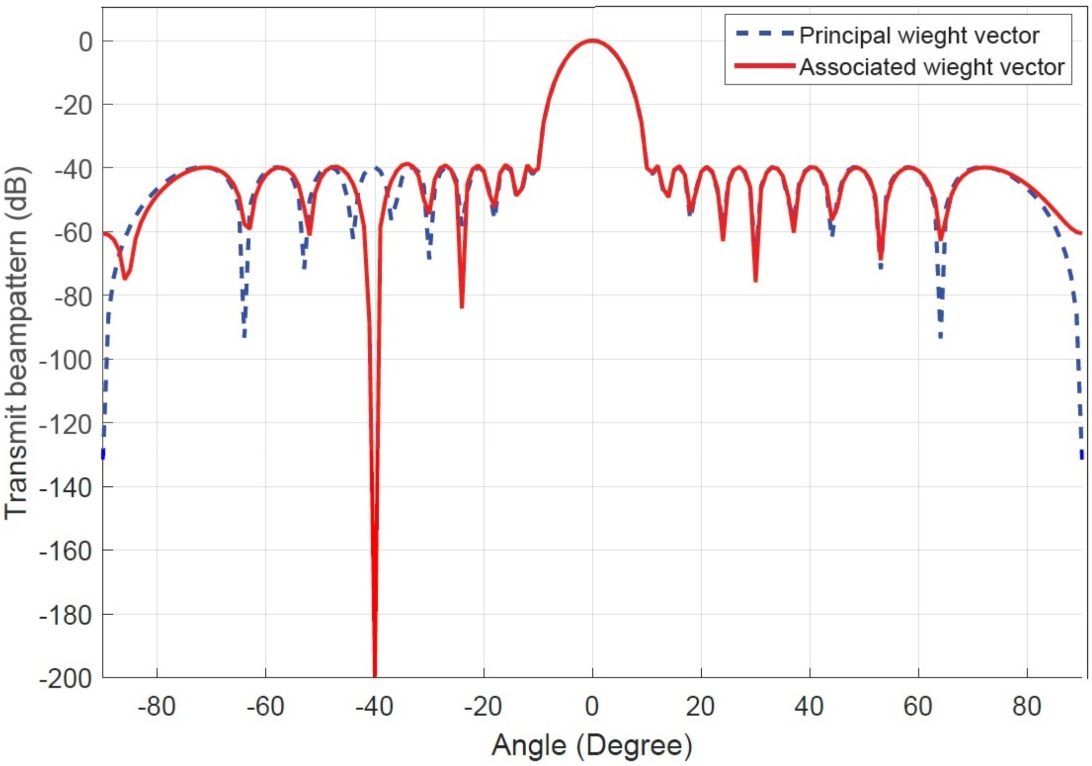

Figure 1. Transmit beampattern versus angle.

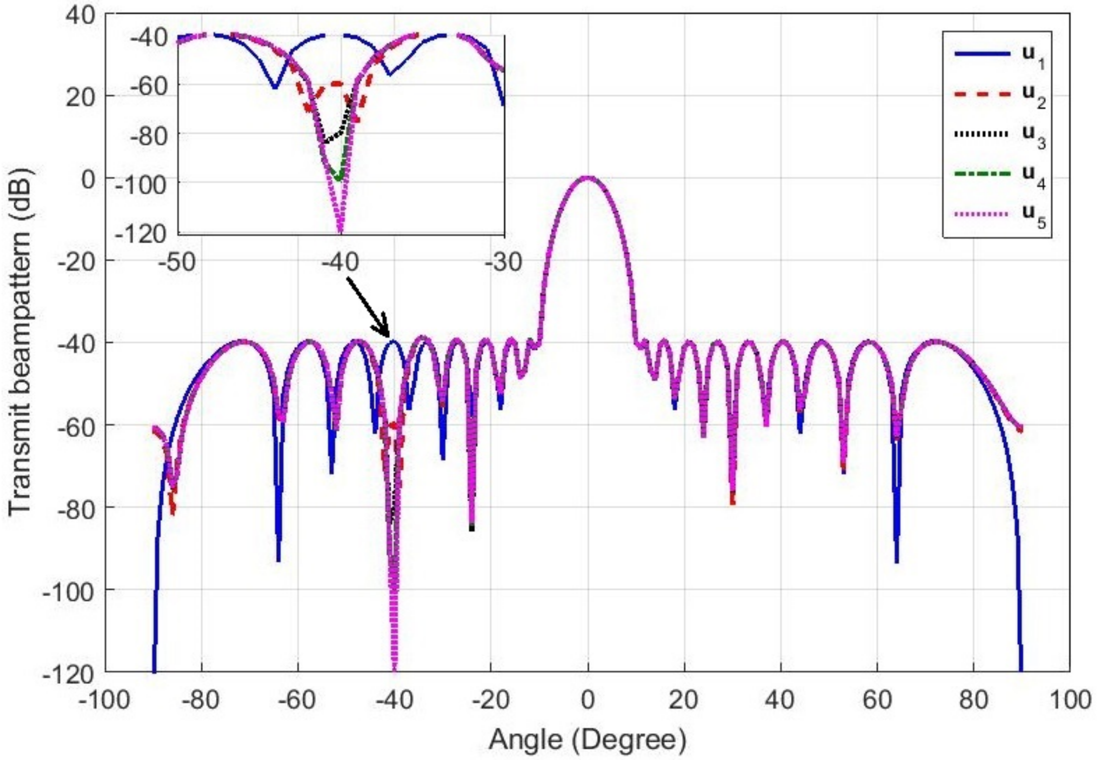

Figure 2. Beampattern versus angle synthesized using (12).

5. CONCLUSIONS

A simple and computationally cheap method for transmit beampattern synthesis which requires designing and storing only two beamforming weight vectors was developed. The proposed method first designs a principal transmit beamforming weight vector based on the requirements dictated by the radar function of the DFRC system. The second weight vectors was obtained by enforcing a deep null towards the intended communication directions. Additional SLLs were realized by simply taking weighted linear combinations of the two available weight vectors. The effectiveness of the proposed method for beampattern synthesis was verified using simulations examples.

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